Instructors: **1.** Dr. Rola Alseidi	**Philadelphia University** **Faculty of Science** **Department of Mathematics** **Midterm Exam**	**Academic Year:** 2022-2023 **Semester:** Second **Date:** 15/05/2023 **Course:** Real Analysis (1) **Duration:** 75 Min

Name: **I.D. Number:**

Question One: [5 points]

Let $S \subseteq \mathbb{R}$, prove that if a number $u \in \mathbb{R}$ has the properties

- $\forall n \in \mathbb{N} : u - \frac{1}{n}$ is not an upper bound S.
- $\forall n \in \mathbb{N} : u + \frac{1}{n}$ is an upper bound of S,

then $supS = u$.

From questions two to five, solve three of them

Question Two: [6 points]

1. Give the exact definition of the limit of a sequence x_n as $n \to \infty$.

2. Use the definition of the limit above to show that

$$\lim_{n\to\infty} \frac{2n+1}{3n+1} = \frac{2}{3}$$

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Question Three: [6 points]

1. State the Monotone Convergence Theorem .

2. Let $x_1 > 1$ and $x_{n+1} = 2 - \dfrac{1}{x_n}$ for $n \in \mathbb{N}$, use the Monotone Convergence Theorem to prove that $\lim\limits_{n\to\infty} x_n = 1$.

Question Four: [6 points]

1. Give the exact definition of Cauchy Sequence.

2. Use the definition of Cauchy sequence to show that $x_n = \dfrac{n}{n+1}$ is a cauchy sequence.

Question Five: [6 points]

1. State and prove The Bolzano Weirstrass Theorem.

2. Use the Bolzano Weirstrass Theorem to show that $\lim_{n\to\infty} \sin(n)$ doesn't exist .

Question Six: [5 points]

Circle True or False. Read each statement carefully before answering.

(a) True False $\lim_{n\to\infty} n^2$ doesn't exist.

(b) True False Every bounded sequence converges.

(c) True False If $\lim_{n\to\infty} x_n = 0$ and y_n is a bounded sequence, then $\lim_{n\to\infty} x_n y_n = 0$.

(d) True False The product of two divergent sequences diverges.

(e) True False If $\lim_{n\to\infty} x_n > 0, \forall n$, then $x > 0$.

Question Seven: [2 points]
Give a counter example for the following statement, explain how your example works.

If X_n is an unbounded sequence, then x_n is properly divergent.